MATH 0120 Business Calculus. Worksheet 2.

University of Pittsburgh, 6W2-Summer 2019

Name:_____

Graph sketching

1. Sketch the graph of the function $f(x) = \dfrac{x^2}{x^2+9}$.
   Make sure to include in your analysis:
   (a) The domain of the function.
   (b) Intercepts.
   (c) Vertical and horizontal asymptotes.
   (d) Intervals of increase and intervals of decrease.
   (e) Critical values, extreme values.
   (f) Intervals of concavity and inflection points.
   (g) Sketch of the graph.

   You can use the first and second derivatives.

$$f'(x) = \frac{18x}{(x^2+9)^2} \qquad f''(x) = \frac{18(9-3x^2)}{(x^2+9)^3}$$

2. Sketch the graph of the function $f(x) = \dfrac{x^2 + x + 1}{x^2}$.
   Make sure to include in your analysis:

   (a) The domain of the function.
   (b) Intercepts.
   (c) Vertical and horizontal asymptotes.
   (d) Intervals of increase and intervals of decrease.
   (e) Critical values, extreme values.
   (f) Intervals of concavity and inflection points.
   (g) Sketch of the graph.

   You can use the first and second derivatives.

$$f'(x) = \frac{-(x+2)}{x^3} \qquad f''(x) = \frac{2x+6}{x^4}$$

3. Sketch the graph of the function $f(x) = \dfrac{x^2}{e^x}$.
   Make sure to include in your analysis:

   (a) The domain of the function.
   (b) Intercepts.
   (c) Vertical and horizontal asymptotes.
   (d) Intervals of increase and intervals of decrease.
   (e) Critical values, extreme values.
   (f) Intervals of concavity and inflection points.
   (g) Sketch of the graph.

   You can use the first and second derivatives.

$$f'(x) = \frac{x(2-x)}{e^x} \qquad f''(x) = \frac{x^2 - 4x + 2}{e^x}$$

Related rates

(a) If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

(b) A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

Optimization problems

4. If 1200 cm^2 of material is available to make a box with a square and an open top, find the largest possible volume of the box.

5. You operate a tour service that offers the following rates: \$200 per person if 50 people (the minimum number to book the tour) go on the tour. For each additional person, up to a maximum of 80 people total, the rate per person is reduced by \$2. It costs \$6000 (a fixed cost) plus \$32 per person to conduct the tour. How many people does it take to maximize your profit?

6. Rent-A-Reck Incorporate finds that it can rent 60 cars if it chrages \$80 for a weekend. It estimates that for each \$5 price increase it will rent three fewer cars. What price should it charge to maximize its revenue? How many cars will it rent at this price?

7. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

8. What are the dimensions of the lightest open-top right circular cylindrical can that will hold a volume of 1000 cm^3?

Differential equations and applications

9. Find the solution to the differential equation that satisfies the initial condition:

   (a) $\dfrac{dy}{dx} = \dfrac{\ln x}{xy}$, with $y(1) = 2$

   (b) $\dfrac{dx}{dt} = 1 - t + x - tx$, with $x(1) = 3$

10. Biologists stocked a lake with 400 fish and estimated that the carrying capacity to be 10000. The number of fish tripled in the first year.
    (a) Assuming that the size of the fish population satisfies the logistic equation, find an expression for the size of the population after t years.
    (b) How long will it take fr the population to increase to 5000?